General Solution and particular Solution.

Let $f(x, y, y', y'', \ldots y^n) = 0$ be a differential equation where $y' = \frac{dy}{dx}$, $y'' = \frac{d^2y}{dx^2}$, ..., etc. The solution of the differential equation containing n essential arbitrary constants is called the general solution of the differential equation.

If we put particular values to the arbitrary constant(s), it is said to be solutions which has no arbitrary consta and also cannot be obtained from the general solution by any choice of the essential arbitrary constants. Such solutio are called singular solutions.

To find essential arbitrary constants fo the solution to

$$\frac{d^2y}{dx^2} + x = 0$$

$y = A_1 \sin(x + A_2) + B_1 \cos(x + B_2)$ is a solution. but A_1, A_2, B_1 & B_2 are not all essential arbitrary constants, beca a smaller set of arbitrary constants wi suffice.

$$y = A_1 \sin(x + A_1) + B_1 \cos(x + B_2)$$

$$= A_1(\sin x \cos A_1 + \cos x \sin A_1) + B_1(\cos x \cos B_2 - \sin x \sin B_2)$$

$$= (A_1 \cos A_1 - B_1 \sin B_2)\sin x + (A_1 \sin A_1 + B_1 \cos B_2)\cos x$$

$$y = A \sin x + B \cos x$$

where $A = A_1 \cos A_1 - B_1 \sin B_2$

$B = A_1 \sin A_1 + B_1 \cos B_2$

Thus, the essential arbitrary constants are just A & B.
